Twin Study Methods: Intraclass Correlation

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Correlations are commonly computed using the standard Pearson product-moment procedure. This involves assigning one member of a bivariate pair to the X variable and the other member of the pair to the Y variable. This designation can be readily made, for instance, with husband-wife data, where each member of the pair unequivocally belongs to a separate category. With twin data however, the characterization of either member of the pair as X or Y is arbitrary. Many $X - Y$ labellings are possible and as a consequence the correlation value obtained is not unique. Consider for example the number of permutations possible with the 3 unordered pairs $(a, b), (c, d), (e, f)$:

X	ace	ade	acf	adf	bcf	bde	bdf	bce
Y	bdf	bcf	bde	bce	ade	acf	ace	adf

8 different $X - Y$ pairings are possible and from these 8 (in actuality, there are 4 since correlation is symmetric about $X - Y$ and half the pairings are mirror images), values for the correlation coefficient can be calculated. More generally, for n unordered pairs, there are 2^n such pairings and these permutations give rise to a distribution of correlation coefficients.

While it is possible to calculate the mean and confidence limits for this distribution and hence infer a value for the correlation, permutation and resampling methods are computationally intensive. A more standard approach for obtaining a measure for the correlation in situations where we are dealing with unordered pairs is to set up the problem as an analysis of variance (ANOVA) calculation. It is easy to see that if both members of a pair have relatively high values, the mean value for that pair will also be relatively high. Conversely, when both members of a pair have relatively low values, the mean for that pair will be relatively low. Hence, the greater the correlation, the greater will be the variability between the means of the pairs as a proportion of total variability, and the smaller will be the proportion of total variability that exists within the pairs. The degree of relationship can thus be estimated by the proportion of the total variability that is accounted for by between class variance. To distinguish this measure from the Pearson product-moment correlation, we define it as an *intraclass correlation (ICC)* coefficient

$$\rho = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_e^2}. \tag{1}$$

In this formulation, σ_s^2 is the true variance between pairs and σ_e^2 is the pooled variance within the pairs. In order to obtain estimates of these parameters, equation (1) can be cast in an ANOVA framework and σ_s^2 and σ_e^2 reinterpreted in terms of the *mean square*. The mean square in ANOVA parlance is an estimate of population variance based on the variability among a set of measures. σ_e^2 is then simply the mean-square estimate of within-pair variance (MS_{within}) computed in ANOVA. If a group or pair is comprised of k members, then the mean-square estimate of between-pair variance ($MS_{between}$) equals k times σ_s^2, the true component, plus σ_e^2, the within pair error component. This is due to the fact that the individual variances add-up and each mean contains a true component and an error term. In other words,

$$\begin{aligned} MS_{within} &= \sigma_e^2 \\ MS_{between} &= k\sigma_s^2 + \sigma_e^2 \end{aligned}$$

From this we get:

$$\sigma_s^2 = \frac{MS_{between} - MS_{within}}{k}$$

Substituting these in the expression for ICC we have:

$$\begin{aligned} \rho &= \frac{(MS_{between} - MS_{within})/k}{(MS_{between} - MS_{within})/k + MS_{within}} \\ &= \frac{MS_{between} - MS_{within}}{MS_{between} - MS_{within} + kMS_{within}} \\ &= \frac{MS_{between} - MS_{within}}{MS_{between} + (k-1)MS_{within}} \end{aligned}$$

This is the same expression derived by Shrout and Fleiss [1] for a design that corresponds to a one-way ANOVA where each pair is a random effect and the members of each pair are viewed as measurement errors. For twin pairs, $k = 2$, and we get the following expression for ICC:

$$ICC = \frac{MS_{between} - MS_{within}}{MS_{between} + MS_{within}}$$

The intraclass correlation ranges from 1.0 to −1.0. It is large and postive when there is little variation within the pairs but the means between the pairs differ. It is large and negative when the variation within a pair is much greater than that between the pairs.

References

[1] Shrout, Patrick E., Fleiss, Joseph L., "Intraclass Correlations: Uses in Assessing Rater Reliability," *Psychological Bulletin,* **86**(2), (1979): 420-428.