A Comprehensive Riemannian Framework for the Analysis of White Matter Fiber Tracts

Meena Mani[1] Sebastian Kurtek[2]
Christian Barillot[1] Anuj Srivastava[2]

[1]Visages Project, INRIA/IRISA, Rennes, France

[2]Department of Statistics, Florida State University, Tallahassee, USA

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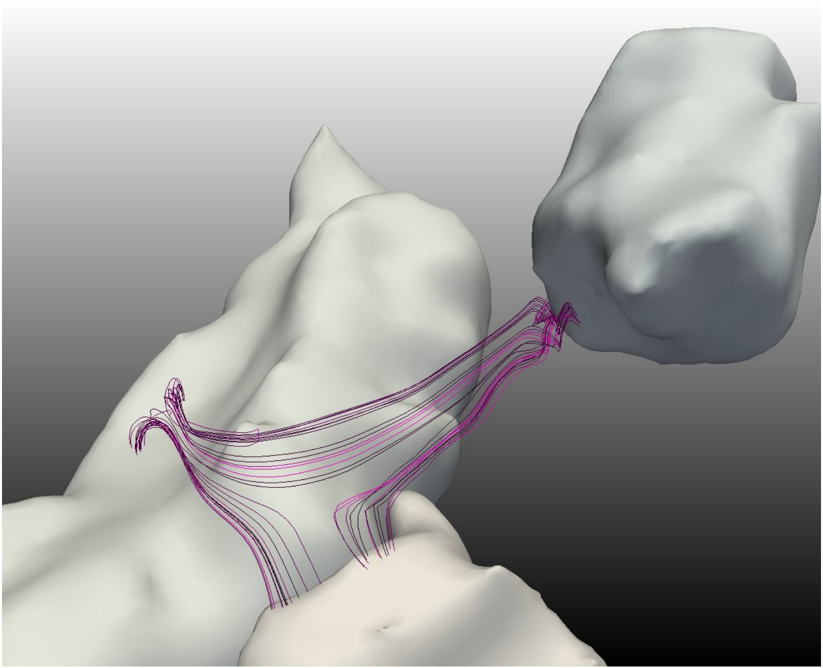

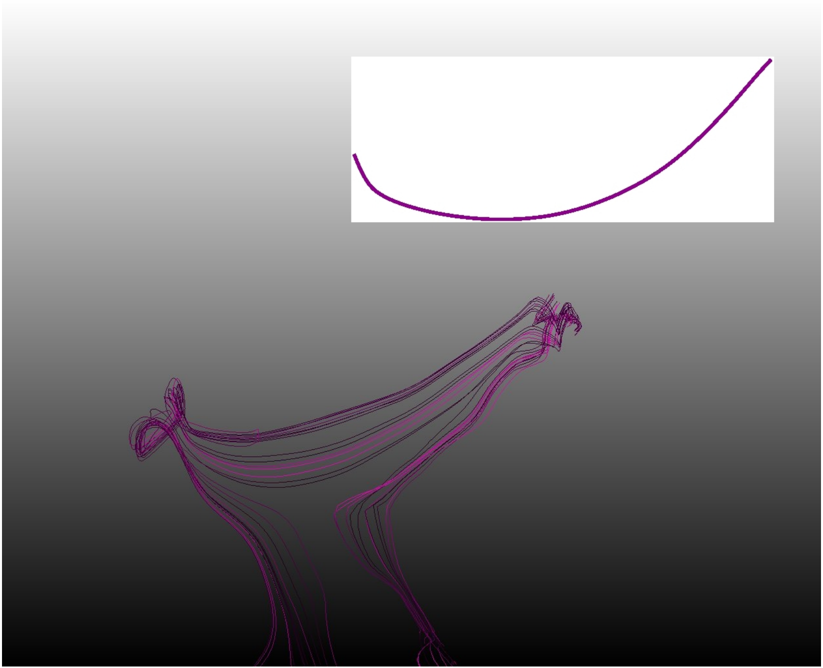

DTI Fiber: Physical Features

position

orientation

scale

shape

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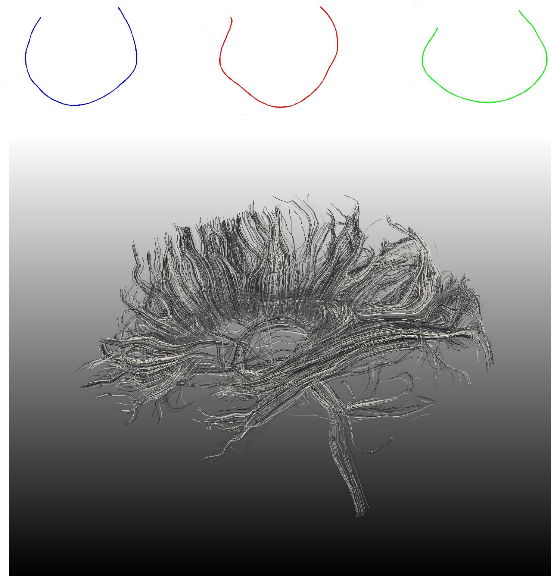

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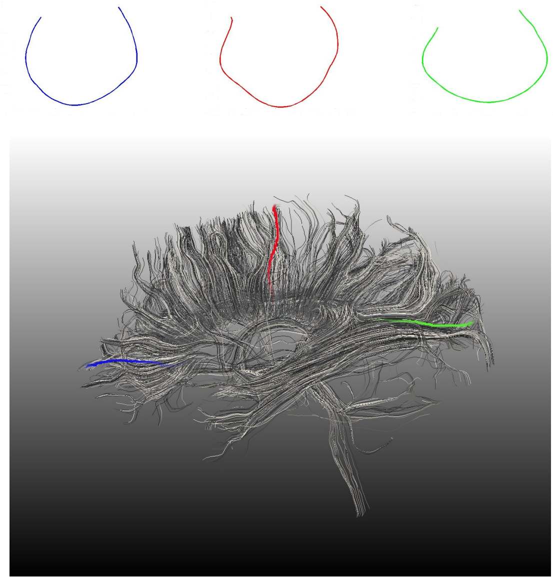

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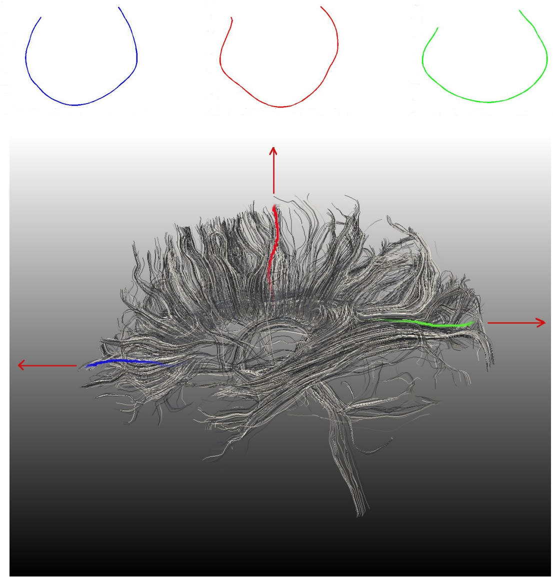

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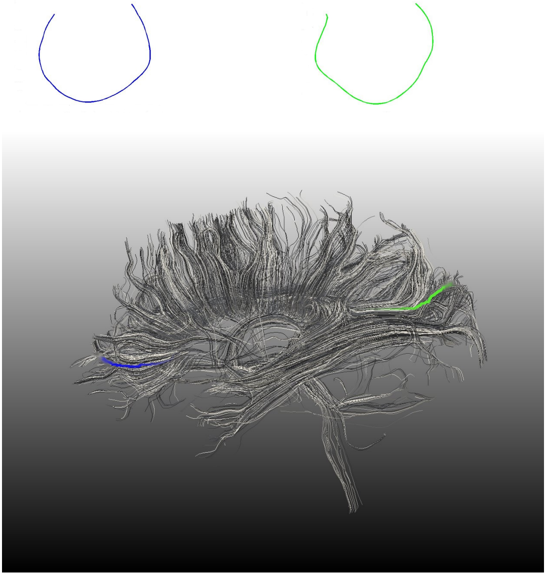

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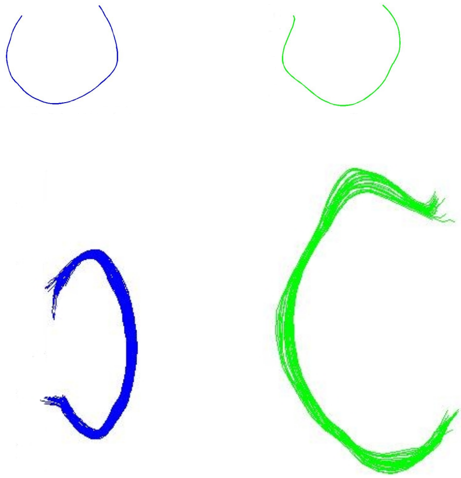

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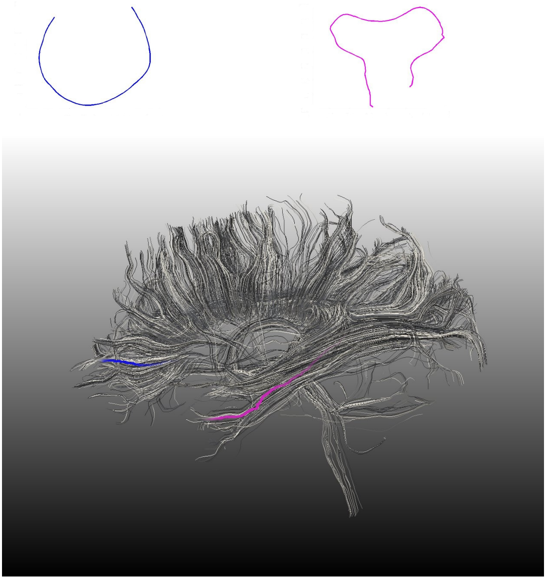

DTI Fiber: Physical Features

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shape

These features either individually or in combination can be used to design metrics and feature spaces

MOTIVATION

White Matter Fiber Analysis

- varied applications
- different goals and perspectives

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White Matter Fiber Analysis

- varied applications
- different goals and perspectives

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- need to design appropriate metrics

Outline

1. Mathematical Framework
   - Fiber Tract Representation
   - $\mathcal{S}_1$: Shape + orientation + scale + position
   - $\mathcal{S}_2$: Shape + orientation + scale
   - $\mathcal{S}_3$: Shape + scale
   - $\mathcal{S}_4$: Shape + orientation
   - $\mathcal{S}_5$: Shape

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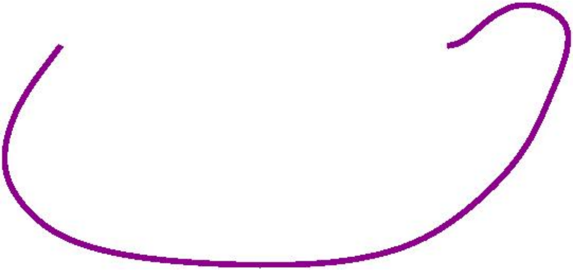
3. Statistics

Fiber Tract Representation

Let $\beta : [0, 1] \rightarrow \mathbb{R}^3$ be a parameterized curve



Comparing Curves

Common Technique:

$\mathbb{L}^2$ distance: $\|\beta_1 - \beta_2\| = \sqrt{\int_0^1 \|\beta_1(t) - \beta_2(t)\|^2 dt}$
For example, methods based on Fourier representation use this metric.

Problem: This metric is not invariant to reparametrization.
Let $\gamma : [0, 1] \to [0, 1]$ be a diffeomorphism (γ acts as a re-parameterization function). A curve β is re-parameterized by γ as $(\beta \circ \gamma)(t) = \beta(\gamma(t))$.
It can be shown that $\|\beta_1 \circ \gamma - \beta_2 \circ \gamma\| \neq \|\beta_1 - \beta_2\|$ in general.

Solution: Seek a new representation that preserves $\mathbb{L}^2$ distance under re-parameterization. The choice of representation depends on the features we want to include in the analysis.

Shape + orientation + scale + position ($\mathcal{S}_1$)

Curve Representation: "h"-function

$$h(t) = \sqrt{\|\dot{\beta}(t)\|}\beta(t) \ , \quad h : [0,1] \rightarrow \mathbb{R}^3$$

The h-function of a re-parameterized curve is: $(h, \gamma) \equiv h(\gamma(t))\sqrt{\dot{\gamma}(t)}$

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Motivation: $\|(h_1,\gamma) - (h_2,\gamma)\| = \|h_1 - h_2\|$ for all γ

Curve-Comparison/Distance function:

$$\gamma^* = \underset{\gamma\in\Gamma}{\operatorname{argmin}}\left(\|h_1 - (h_2,\gamma)\|\right), \quad d_a(\beta_1,\beta_2) = \|h_1 - (h_2,\gamma^*)\|$$

d_a is invariant to re-parameterizations of β_1 and β_2

Geodesic path or Deformation: Straight line

$$\psi(\tau) = (1-\tau)h_1 + \tau(h_2,\gamma^*)$$

Shape + orientation + scale ($\mathcal{S}_2$)

Curve Representation: "q"-function or square-root velocity function

$$q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$$

Since q is defined using $\dot{\beta}$, it is invariant to the translation of β

The q-function of a re-parameterized curve is: $(q, \gamma) \equiv q(\gamma(t))\sqrt{\dot{\gamma}(t)}$

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Motivation: $\|(q_1, \gamma) - (q_2, \gamma)\| = \|q_1 - q_2\|$ for all γ

Curve-Comparison/Distance function:

$$\gamma^* = \operatorname*{argmin}_{\gamma \in \Gamma} \left(\|q_1 - (q_2, \gamma)\| \right), \quad d_b(\beta_1, \beta_2) = \|q_1 - (q_2, \gamma^*)\|$$

Geodesic path or Deformation: Straight line

$$\psi(\tau) = (1 - \tau)q_1 + \tau(q_2, \gamma^*)$$

Shape + scale ($\mathcal{S}_3$)

We remove rotation from the representation as follows:

Curve Representation: q-function

$$q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$$

Curve Alignment:

$$(O^*, \gamma^*) = \underset{O \in SO(3), \gamma \in \Gamma}{\operatorname{argmin}} \left(\|q_1 - O(q_2, \gamma)\|\right)$$

Distance function:

$$d_d(\beta_1, \beta_2) = \|q_1 - O^*(q_2, \gamma^*)\|$$

Geodesic path: Straight line

$$\psi(\tau) = (1 - \tau)q_1 + \tau(O^* q_2, \gamma^*)$$

Shape + orientation ($\mathcal{S}_4$)

Keep orientation but remove scale by rescaling all curves

Curve Representation: $q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$

Distance function: Arc-Length on a sphere

$$d_c(\beta_1, \beta_2) = \min_{\gamma \in \Gamma} \left(\cos^{-1}(\int_0^1 \langle (q_1, \gamma)(t), (q_2, \gamma)(t) \rangle \, dt) \right)$$

Geodesic path: Great circle!

$$\psi(\tau) = \frac{1}{\sin(\theta)} [\sin(\theta - \tau\theta) q_1 + \sin(\tau\theta)(q_2, \gamma^*)]$$

where $\theta = d_c(\beta_1, \beta_2)$

Shape ($\mathcal{S}_5$)

Now we remove all variables: rotation, translation and scale

Curve Representation: $q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$

Curve Alignment:

$$(O^*, \gamma^*) = \underset{O \in SO(3), \gamma \in \Gamma}{\mathrm{argmin}} \left(\cos^{-1}(\int_0^1 \langle (q_1, \gamma)(t), O(q_2, \gamma)(t) \rangle \, dt) \right)$$

Distance function:

$$d_e(\beta_1, \beta_2) = \cos^{-1}(\int_0^1 \langle (q_1, \gamma)(t), O^*(q_2, \gamma)(t) \rangle \, dt)$$

Geodesic path: Great circle!

$$\psi(\tau) = \frac{1}{\sin(\theta)} [\sin(\theta - \tau\theta) q_1 + \sin(\tau\theta)(O^* q_2, \gamma^*)]$$

for $\theta = d_e(\beta_1, \beta_2)$

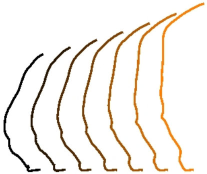
In Summary

Manifold	function representation	pre-shape space	shape space
shape + orientation + scale + position	$h(t) = \sqrt{\|\dot{\beta}(t)\|}\beta(t)$	$\mathcal{C}_1$ $\mathbb{L}^2$ space	$\mathcal{S}_1 = \mathcal{C}_1/(\Gamma)$
shape + orientation + scale	$q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$	$\mathcal{C}_2$ $\mathbb{L}^2$ space	$\mathcal{S}_2 = \mathcal{C}_2/(\Gamma)$
shape + scale	$q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$	$\mathcal{C}_2$ $\mathbb{L}^2$ space	$\mathcal{S}_3 = \mathcal{C}_2/(\Gamma \times SO(3))$
shape + orientation	$q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$	$\mathcal{C}_3$ hypersphere	$\mathcal{S}_4 = \mathcal{C}_3/(\Gamma)$
shape	$q(t) = \frac{\dot{\beta}(t)}{\sqrt{\|\dot{\beta}(t)\|}}$	$\mathcal{C}_3$ hypersphere	$\mathcal{S}_5 = \mathcal{C}_3/(\Gamma \times SO(3))$

Geodesic Paths

Evolution of one curve into another

$\mathcal{S}_2$: shape+orientation+scale



$\mathcal{S}_3$: shape+scale

$\mathcal{S}_4$: shape+orientation

$\mathcal{S}_5$: shape

Application: Clustering Fibers in the Corpus Callosum

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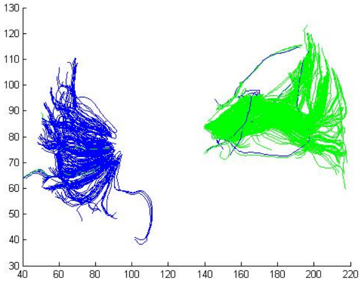
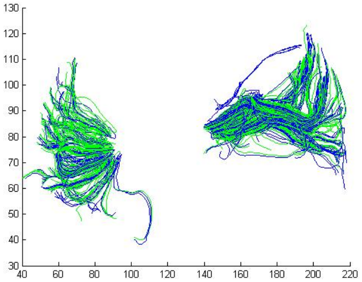


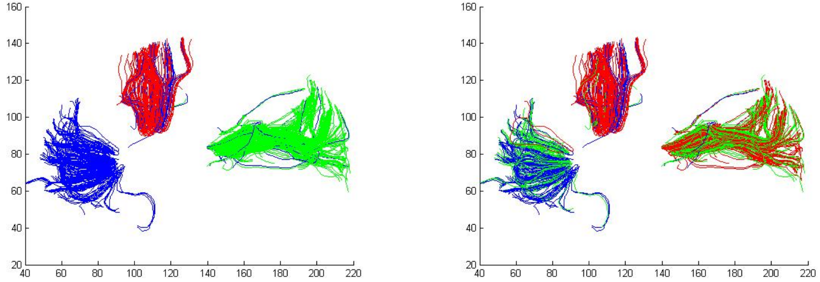
(a) shape+orientation+scale (d_b)



(b) shape (d_e)

Clustering the genu and splenium, the anterior and posterior sections of the CC. Here, shape information alone (b) is not adequate for clustering.

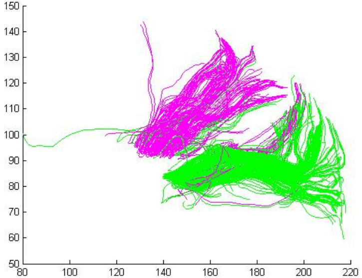
Application: Clustering Fibers in the Corpus Callosum



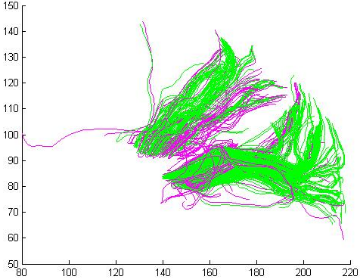
(a) shape+orientation (d_c)   (b) shape+orientation+scale (d_b)

Clustering of the genu, corpus and splenium, the anterior, middle and posterior sections of the CC. Including the scale information results in poorer clustering (b).

Application: Clustering Fibers in the Corpus Callosum



(a) shape+orientation (d_c)



(b) shape+scale (d_d)

Clustering the isthmus and splenium, the posterior CC

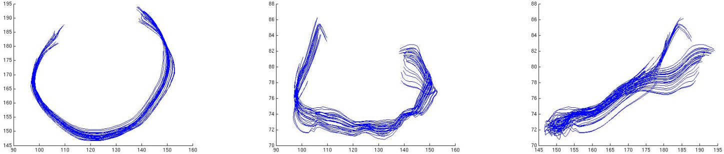
Statistics

- Summary statistics
- Statistical inference
  - Stochastic models
  - Hypothesis testing

Statistics

Mean Curves of a DTI Fiber Bundle: Splenium

fiber bundle



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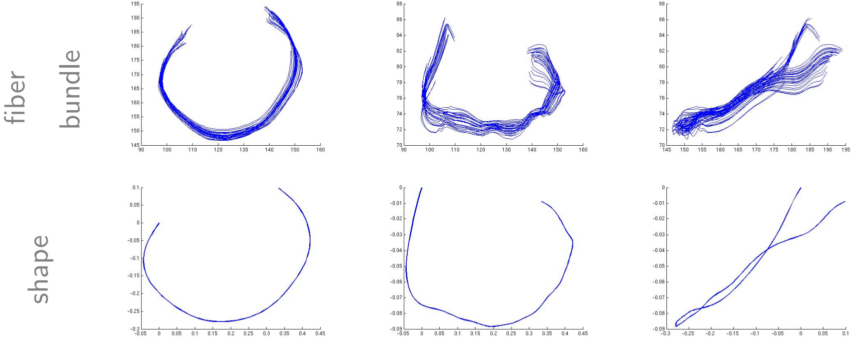

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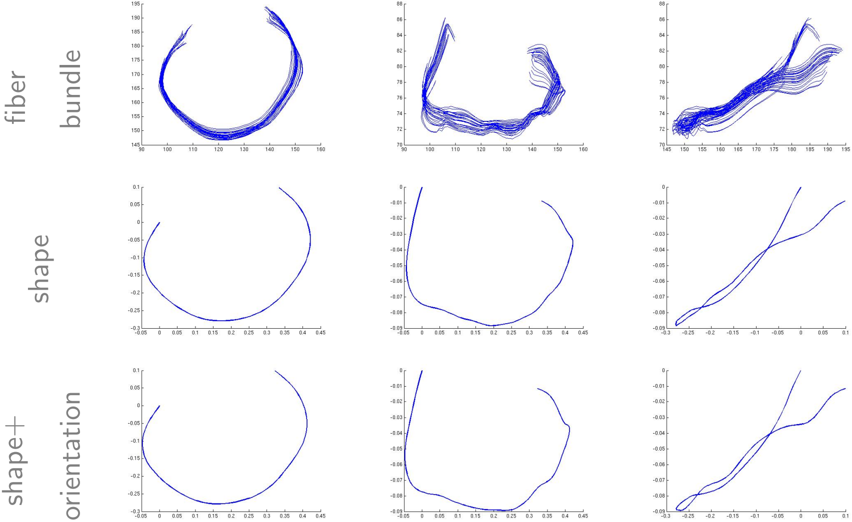
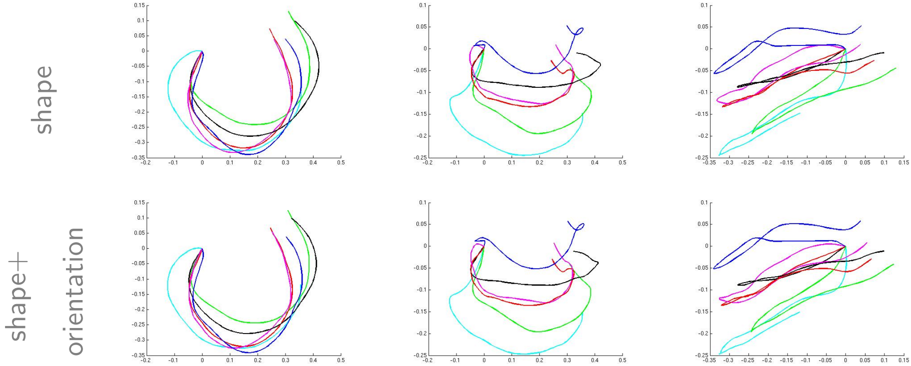


Statistics

Mean Curves of a Population: Spleniums of 6 Subjects



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Search terms: Anuj Srivastava, FSU, Publications

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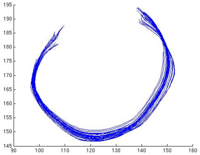

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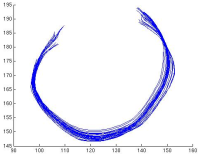
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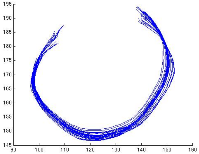
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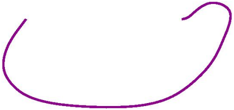

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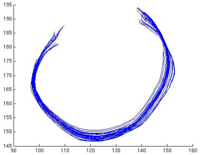
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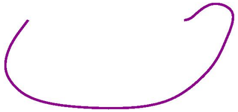

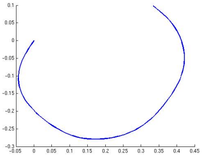
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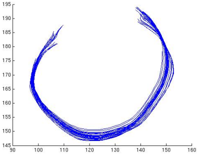
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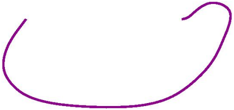

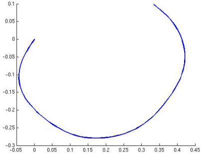
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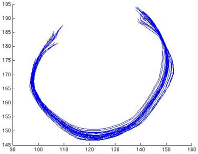
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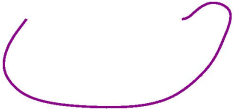

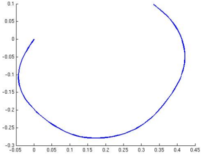
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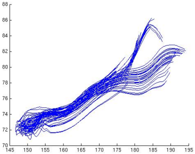
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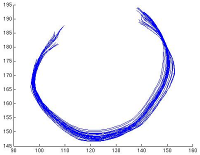

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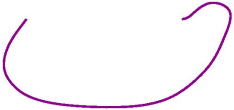

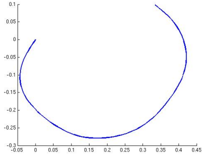

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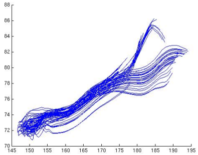

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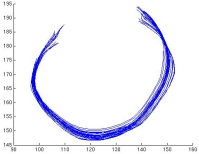

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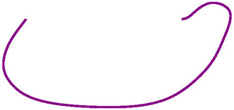

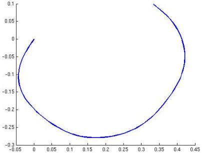

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0
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-0.1
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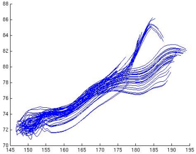

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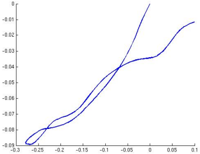

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-0.04
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-0.06
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